CHAPTER VII

Einstein: The Thinking That Led to the Theory of Relativity

What were the decisive steps in the development of Einstein's theory of relativity? Although this is quite a task, I shall try to make them clear to the reader. A number of issues, such as the question of the ether, the relations to the principles of Galilean "relativity," will be excluded from the discussion. The field which Einstein faced in this gigantic process of thought was very large, since it involved most of the fundamental topics of modern physics, difficult issues unfamiliar to those not acquainted with the complexity of modern physics. Although the following sketch will of necessity be condensed, I hope the reader will gain an insight into the nature of the decisive steps.

Those were wonderful days, beginning in 1916, when for hours and hours I was fortunate enough to sit with Einstein, alone in his study, and hear from him the story of the dramatic developments which culminated in the theory of relativity. During those long discussions I questioned Einstein in great detail about the concrete events in his thought. He described them to me, not in generalities, but in a discussion of the genesis of each question.

Einstein's original papers give his results. They do not tell the story of his thinking. In the course of one of his books he did report some steps in the process. I have quoted him in the proper places in this chapter.

Act I. The Beginning of the Problem

The problem started when Einstein was sixteen years old, a pupil in the Gymnasium (Aarau, Kantonschule). He was not an especially good student, unless he did productive work on his own account. This he did in physics and mathematics, and consequently he knew more about these subjects than his classmates. It was then that the great problem really started to trouble him. He was intensely concerned with it for seven years; from the moment, however, that he came to question the customary concept of time (see Act VII), it took him only five weeks to write his paper on relativity—although at this time he was doing a full day's work at the Patent Office.

The process started in a way that was not very clear, and is therefore difficult to describe—in a certain state of being puzzled. First came such questions as: What if one were to run after a ray of light? What if one were riding on the beam? If one were to run after a ray of light as it travels, would its velocity thereby be decreased? If one were to run fast enough, would it no longer move at all? . . . To young Einstein this seemed strange.

The same light ray, for another man, would have another velocity. What is "the velocity of light"? If I have it in relation to something, this value does not hold in relation to something else which is itself in motion. (Puzzling to think that under certain conditions light should go more quickly in one direction than another.) If this is correct, then consequences would also have to be drawn with reference to the earth, which is moving. There would then be a way of finding out by experiments with light whether one is in a moving system! Einstein's interest was captured by this; he tried to find methods by which it would be possible to establish or to measure the movement of the earth—and he learned only later that physicists had already made such experiments. His wish to design such experiments was always accompanied by some doubt that the thing was really so; in any case, he felt that he must try to decide.

He said to himself: "I know what the velocity of a light ray is in relation to a system. What the situation is if another system is taken

into account seems to be clear, but the consequences are very puzzling."

Act II. Light Determines a State of Absolute Rest?

Would operations with light lead to conclusions different in this respect from conclusions from mechanical operations? From the point of view of mechanics there seems to be no absolute rest; from the point of view of light there does seem to be. What of the velocity of light? One must relate it to something. Here the trouble starts. Light determines a state of absolute rest? However, one does not know whether or not one is in a moving system. Young Einstein had reached some kind of conviction that one cannot notice whether or not one is in a moving system; it seemed to him deeply founded in nature that there is no "absolute movement." The central point here became the conflict between the view that light velocity seems to presuppose a state of "absolute rest" and the absence of this possibility in the other physical processes.

Back of all this there had to be something that was not yet grasped, not yet understood. Uneasiness about this characterized young Einstein's state of mind at this time.

When I asked him whether, during this period, he had already had some idea of the constancy of light velocity, independent of the movement of the reference system, Einstein answered decidedly: "No, it was just curiosity. That the velocity of light could differ depending upon the movement of the observer was somehow characterized by doubt. Later developments increased that doubt." Light did not seem to answer when one put such questions. Also light, just as mechanical processes, seemed to know nothing of a state of absolute movement or of absolute rest. This was interesting, exciting.

Light was to Einstein something very fundamental. At the time of his studies at the Gymnasium, the ether was no longer being thought of as something mechanical, but as "the mere carrier of electrical phenomena."

Act III. Work on the One Alternative

Serious work started. In the Maxwell equations of the electromagnetic field, the velocity of light plays an important role; and it is constant. If the Maxwell equations are valid with regard to one system, they are not valid in another. They would have to be changed. When one tries to do so in such a way that the velocity of light is not assumed to be constant, the matter becomes very complicated. For years Einstein tried to clarify the problem by studying and trying to change the Maxwell equations. He did not succeed in formulating these equations in such a way as to meet the difficulties satisfactorily. He tried hard to see clearly the relation between the velocity of light and the facts of movement in mechanics. But in whatever way he tried to unify the question of mechanical movement with the electromagnetic phenomena, he got into difficulties. One of his questions was: What would happen to the Maxwell equations and to their agreement with the facts if one were to assume that the velocity of light depends on the motion of the source of the light?

The conviction grew that in these respects the situation with regard to light could not be different from the situation with regard to mechanical processes (no absolute movement, no absolute rest). What took him so much time was this: he could not doubt that the velocity of light is constant and at the same time get a satisfactory theory of electromagnetic phenomena.

Act IV. Michelson's Result and Einstein

The famous Michelson experiment confronted physicists with a disconcerting result. If you are running away from a body that is rushing toward you, you will expect it to hit you somewhat later than if you are standing still. If you run toward it, it will hit you earlier. Michelson did just this in measurements of the velocity of light. He compared the time light takes to travel in two pipes if these pipes meet at right angles to each other, and if one lies in the direction of the movement of the earth, while the other is vertical to

¹Cf. below, Act IX.

The layman, not acquainted with modern physics, will not be able to follow the topics under II and III in my brief formulations. While these topics were important in the process, it does not seem absolutely necessary to understand them fully in order to follow the later steps within the positive solution. The lay reader may therefore pass on to Act IV.

it. Since the first pipe, in its lengthwise direction, is moving with the movement of the earth, the light traveling in it ought to reach the receding end of this pipe later than the light in the other pipe reaches its end.



The arrows above indicate the directions in which the light is traveling. The movement of the earth, and so of the whole apparatus is toward the right.

Actually the arrangement was more intricate. There was at the vertex angle of the two pipes a common mirror, and there were mirrors at the ends. In both pipes light rays from a common source were moving forward and backward, reflected by the mirrors. The difference in time was to be measured by means of an interference effect at the common mirror. (The reader may be inclined to assume that, with the light rays moving forward and backward, the difference introduced by the movement of the earth would be canceled. This is not the case, as some mathematical deliberation will show.) The difference could not escape observation because the measurement by interference was sufficiently delicate to reveal the amount as established by mathematical analysis.

No difference was found. The experiment was repeated, and the negative result was clearly confirmed.

The result of the Michelson experiment in no way fitted the fundamental ideas of the physicists. In fact the result contradicted all their reasonable expectations.

For Einstein, Michelson's result was not a fact for itself. It had its place within his thoughts as they had thus far developed. Therefore, when Einstein read about these crucial experiments made by physicists, and the finest ones made by Michelson, their results were no surprise to him, although very important and decisive. They seemed to confirm rather than to undermine his ideas. But the

matter was not yet entirely cleared up. Precisely how does this result come about? The problem was an obsession with Einstein although he saw no way to a positive solution.

Act V. The Lorentz Solution

Not only Einstein was troubled; many physicists were. Lorentz, the famous Dutch physicist, had developed a theory which formulated mathematically what had occurred in the Michelson experiment. In order to explain this fact it seemed necessary to him, as it had to Fitzgerald, to introduce an auxiliary hypothesis: he assumed that the entire apparatus used in the measurement underwent a small contraction in the direction of the earth's motion. According to this theory, the pipe in the direction of the movement of the earth was changed in length, while the other pipe suffered only a change in width and the length remained unaffected. The contraction had to be assumed to be just the amount needed to compensate for the effect of the earth's motion on the traveling of the light. This was an ingenious hypothesis.

There was now a fine, positive formula, determining the Michelson results mathematically, and an auxiliary hypothesis, the contraction. The difficulty was "removed." But for Einstein the situation was no less troublesome than before; he felt the auxiliary hypothesis to be a hypothesis ad hoc, which did not go to the heart of the matter.

Act. VI. Re-examination of the Theoretical Situation

Einstein said to himself: "Except for that result, the whole situation in the Michelson experiment seems absolutely clear; all the factors involved and their interplay seem clear. But are they really clear? Do I really understand the structure of the whole situation, especially in relation to the crucial result?" During this time he was often depressed, sometimes in despair, but driven by the strongest vectors.

In his passionate desire to understand or, better, to see whether the situation was really clear to him, he faced the essentials in the Michelson situation again and again, especially the central point: the measurement of the speed of light under conditions of movement of the whole set in the crucial direction.

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This simply would not become clear. He felt a gap somewhere without being able to clarify it, or even to formulate it. He felt that the trouble went deeper than the contradiction between Michelson's actual and the expected result.

He felt that a certain region in the structure of the whole situation was in reality not as clear to him as it should be, although it had hitherto been accepted without question by everyone, including himself. His proceeding was somewhat as follows: there is a time measurement while the crucial movement is taking place. "Do I see clearly," he asked himself, "the relation, the inner connection between the two, between the measurement of time and that of movement? Is it clear to me how the measurement of time works in such a situation?" And for him this was not a problem with regard to the Michelson experiment only, but a problem in which more basic principles were at stake.

Act. VII. Positive Steps toward Clarification

It occurred to Einstein that time measurement involves simultaneity. What of simultaneity in such a movement as this? To begin with, what of simultaneity of events in different places?

He said to himself: "If two events occur in one place, I understand clearly what simultaneity means. For example, I see these two balls hit the identical goal at the same time. But . . . am I really clear about what simultaneity means when it refers to events in two different places? What does it mean to say that this event occurred in my room at the same time as another event in some distant place? Surely I can use the concept of simultaneity for different places in the same way as for one and the same place—but can I? Is it as clear to me in the former as it is in the latter case? . . . It is not!"

For what now followed in Einstein's thinking we can fortunately report paragraphs from his own writing.2 He wrote them in the form of a discussion with the reader. What Einstein here says to the reader is similar to the way his thinking proceeded: "Lightning strikes in two distant places. I assert that both bolts struck simultaneously. If now I ask you, dear reader, whether this assertion makes sense, you will answer, 'Yes, certainly'. But if I urge you to explain to me more clearly the meaning of this assertion, you will find after some deliberation that answering this question is not as simple as it at first appears.

"After a time you will perhaps think of the following answer: 'The meaning of the assertion is in itself clear and needs no further clarification. It would need some figuring out, to be sure, if you were to put me to the task of deciding by observation whether in a concrete case the two effects were actually simultaneous or not."

I now insert an illustration which Einstein offered in a discussion. Suppose somebody uses the word "hunchback." If this concept is to have any clear meaning, there must be some way of finding out whether or not a man has a hunched back. If I could conceive of no possibility of reaching such a decision, the word would have no real meaning for me.

"Similarly," Einstein continued, "with the concept of simultaneity. The concept really exists for the physicist only when in a concrete case there is some possibility of deciding whether the concept is or is not applicable. Such a definition of simultaneity is required, therefore, as would provide a method for deciding. As long as this requirement is not fulfilled, I am deluding myself as physicist (to be sure, as non-physicist too!) if I believe that the assertion of simultaneity has a real meaning. (Until you have truly agreed to this, dear reader, do not read any further.)

"After some deliberation you may make the following proposal to prove whether the two shafts of lightning struck simultaneously. Put a set of two mirrors at an angle of 90° to each other (\sqrt{)}, at the exact halfway mark between the two light effects, station yourself in front of them, and observe whether or not the light effects strike the mirrors simultaneously."

Simultaneity in distant places here gets its meaning by being based on clear simultaneity in an identical place.3

² A. Einstein, Über die spezielle und die allgemeine Relativitätstheorie (Braunschweig, 1916) pp. 14 ff. (Editors' note. The passages from Einstein quoted in this chapter were translated by Professor Wertheimer.)

³ This involves other problems which we shall here ignore. The reader is referred to Einstein. Über die spezielle und allgemeine Relativitätstheorie, pp. 15-16.

All these steps came not by way of isolated clarification of this special question, but as part of the attempt to understand the inner connection that was mentioned above, the problem of the measurement of speed during the crucial movement. In the mirror situation this means simply: what happens if, in the time during which the light rays approach my mirrors, I move with them, away from one source of light and toward the other. Obviously, if the two events appeared simultaneous to a man at rest they would not then appear so to me, who am moving with my mirrors. His statement and mine must differ. We see then that our statements about simultaneity involve essentially reference to movement of the observer. If simultaneity in distant places is to have real meaning, I must explicitly take into account the question of movement, and in comparing my judgments with those of another observer, I have to take into account the relative movement between him and me. When dealing with "simultaneity in different places" I must refer to the relative movement of the observer.

I repeat: suppose that I with my mirrors am traveling in a train going in a straight line at a constant velocity. Two shafts of lightning strike in the distance, one near the engine, the other near the rear end of the train; my double mirror being right in the middle between the two. As a passenger I use the train as my frame of reference, I relate these events to the train. Let us assume that just at the critical moment when the lightning strikes, a man is standing beside the tracks, likewise with double mirrors, and that his place at that moment coincides with mine. What would my observations be and what would his be?

"If we say that the bolts of lightning are simultaneous with regard to the tracks, this now means: the rays of light coming from two equidistant points meet simultaneously at the mirrors of the man on the track. But if the place of my moving mirrors coincides with his mirrors at the moment the lightning strikes, the rays will not meet exactly simultaneously in my mirrors because of my movement.

"Events which are simultaneous in relation to the track are not simultaneous in relation to the train, and vice versa. Each frame of reference, each system of coordinates therefore has its special time;

a statement about a time has real meaning only when the frame of reference is stated, to which the assertion of time refers."4

It has always seemed simple and clear that a statement about the "time difference" between two events is a "fact," independent of other factors, such as movement of the system. But, in actual fact, is not the thesis that "the time difference between two events is independent of the movement of the system" an arbitrary assumption? It did not hold, as we saw, for simultaneity in different places, and therefore it cannot hold even for the length of a second. To measure a time interval, we must use a clock or the equivalent of a clock, and look for certain coincidences at the beginning and at the end of the interval. Therefore the trouble with simultaneity is involved. We cannot dogmatically assume that the time which a certain event takes in relation to the train is the same as the time in relation to the track.

This applies also to the measurement of distances in space! If I try to measure exactly the length of a car by marking its end points on the roadbed, I must take care, when I have made my mark at one end, that the car does not move before I come to the other end! Unless I have explicitly given attention to this possibility, my measurements will be misleading.

I must therefore conclude that in every such measurement reference must be made to the movement of the system. For the observer within the moving system will get results which differ from those of an observer in another frame of reference. "Every system has its special time and space values. A time or space judgment has sense only if we know the system with reference to which the judgment was made." We must change the old view: the measurements of time intervals and of distances in space are *not* independent of the conditions of movement of the system in relation to the observer.

The old view had been a time-honored "truth." Einstein, seeing that it was questionable, came to the conclusion that space and time measurements depend on the movement of the system.

Act VIII. Invariants and Transformation

What followed was determined by two vectors which simultaneously tended toward the same question.

⁴ A. Einstein, op. cit., pp. 31-32.

1. The system of reference may vary; it can be chosen arbitrarily. But in order to reach physical realities, I have to get rid of such arbitrariness. The basic laws must be independent of arbitrarily chosen co-ordinates. If one wants to get a description of physical events, the basic laws of physics must be invariant with regard to such changes.

Here it becomes clear that one might adequately call Einstein's theory of relativity just the opposite, an absolute theory.

2. Insight into the interdependence of time measurement and movement is certainly not enough in itself. What is now needed is a transformation formula that answers this question: "How does one find the place and time values of an event in relation to one moving system, if one knows the places and times as measured in another? Or better, how does one find the transformation from one system to another when they move in relation to each other?"

What would be the direct way? In order to proceed realistically, I would have to base the transformation on an assumption with regard to some physical realities which could be used as invariants.

The reader may think back to an old historical situation. Physicists in past ages tried to construct a *perpetuum mobile*. After many attempts which did not succeed, the question suddenly arose: how would physics look if nature were basically such as to make a *perpetuum mobile* impossible? This involved an enormous change, which recentered the whole field.

Similarly there arose in Einstein the following question, which was inspired by his early ideas mentioned in Acts II and III: How would physics look if, by nature, measurements of the velocity of light would under all conditions have to lead to the identical value? Here is the needed invariant! (Thesis of the basic constancy of the velocity of light.)

In terms of the desired transformation, this means: "Can a relation between the place and time of events in systems which move linearly to each other be so conceived that the velocity of light becomes a constant?"

Eventually Einstein reached the answer: "Yes!" The answer consisted of concrete and definite transformation formulas for distances

in time and space, formulas that differed characteristically from the old Galilean transformation formulas.

3. In the discussions I had with Einstein in 1916 I put this question to him: "How did you come to choose just the velocity of light as a constant? Was this not arbitrary?"

Of course it was clear that one important consideration was the empirical experiments which showed no variation in the velocity of light. "But did you choose this arbitrarily," I asked, "simply to fit in with these experiments and with the Lorentz transformation?" Einstein's first reply was that we are entirely free in choosing axioms. "There is no such difference as you just implied," he said, "between reasonable and arbitrary axioms. The only virtue of axioms is to furnish fundamental propositions from which one can derive conclusions that fit the facts." This is a formulation that plays a great role in present theoretical discussions, and about which most theorists seem to be in agreement. But then Einstein himself smilingly proceeded to give me a very nice example of an unreasonable axiom: "One could of course choose, say, the velocity of sound instead of light. It would be reasonable, however, to select not the velocity of just 'any' process, but of an 'outstanding' process . . ." Questions like the following had occurred to Einstein: Is the speed of light perhaps the fastest possible? Is it perhaps impossible to accelerate any movement beyond the speed of light? As velocity increases, progressively greater forces are required to increase it still further. Perhaps the force required to increase a velocity beyond the velocity of light is infinite?

It was marvelous to hear in Einstein's descriptions how these bold questions and expectations had taken shape in him. It was new, unthought of before, that the velocity of light might be the greatest possible velocity, that an attempt to go beyond that limit would require forces infinitely great.

If these assumptions brought clarity into the system, and if they were proved by experiment, then it would make good sense to take the velocity of light as the basic constant. (Cf. the absolute zero of temperature which is reached when the molecular movements in an ideal gas approach zero.)

4. The derivations which Einstein reached from his transformation

formulas showed mathematical coincidence with the Lorentz transformation. The contraction hypothesis had therefore been in the right direction, only now it was no longer an arbitrary auxiliary hypothesis, but the outcome of improved insight, a logically necessary derivation from the improved view of fundamental physical entities. The contraction was not an absolute event, but a result of the relativity of measurements. It was not determined by a "movement in itself which possesses no real sense for us, but only by a movement with reference to the chosen observation system."

Act IX. On Movement, on Space, a Thought Experiment

The last statement throws new light on the changes in thinking which were already involved in the earlier steps. "By the motion of a body we always mean its change of position in relation to a second body," to a framework, or a system. If there is one body alone, it has no sense to ask or to try to state whether it is moving or not. If there are two, we can state only whether they are approaching or moving away from each other; but, so long as there are only two, it has no sense to ask, or to try to state, whether one is turning around the other; the essential in movement is change of position in relation to another object, a framework, or a system.

But is there not *one* outstanding system in regard to which there is *absolute* movement of a body, "the" space (Newtonian space, the space of the ether), the box in which all movement takes place?

Here I may mention something that happened not just at this point in the development of the process, but may illustrate what was really going on. It transcends the problems of the special theory of relativity: is there no proof of the reality of such an outstanding system? A famous experiment of Newton's had been used as proof: When a sphere of oil rotates it becomes flattened. This is a real, physical, observable fact, apparently caused by an "absolute" movement.

But is this really a demonstration of such an absolute movement? It seems so certainly; but is it actually, if we think it through? In reality we have not a body moving alone in absolute space, but a body that moves within our fixed-star firmament. Is the flattening of that sphere perhaps an outcome of the movement of the sphere

relative to the surrounding stars? What would happen if we took a very huge iron wheel, with a small hole at the center, if we suspended in this hole a little sphere of oil, and then rotated the wheel? Perhaps the little sphere would again become flattened. Then the flattening would have nothing to do with the rotation in an absolute space box; rather it would be determined by the systems moving in relation to each other, the big wheel or the firmament on the one hand and the little sphere of oil on the other.

Of course rotation already transcends the region of the so-called special relativity of Einstein. It became basic in the problem of the general theory of relativity.

Act X. Questions for Observation and Experiment

Einstein is at heart a physicist. Thus all these developments aimed at real, concrete, experimental problems. As soon as he reached clarification he concentrated on the point: "Is it possible to find crucial physical questions to be answered in experiments that will decide whether these new theses are 'true'; whether they fit facts better, give better predictions of physical events than the old theses?"

He found a number of such crucial experiments, some of which physicists could and later did carry out.

II

In actual fact, the problem leads on: it led in Einstein's mind to the problems of the general theory of relativity. But let us stop with the story here and ask ourselves: What were the decisive characteristics of this thinking?

The physicist is interested in the relation of Einstein's theory to established facts, in experimental proof, in the consequences for further development, in the mathematical formulas which follow from the theory of relativity in the various parts of physics.

The theorist of knowledge is interested in the ideas of space, time, and matter, in the "relativistic" character of the theory (with all the wrong consequences in the direction of philosophical, sociological, or ethical relativism drawn by others), in the problem of

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"testability" which played such an important role in Einstein's dealing with simultaneity (and later in the developments of operationalism).

The psychologist, who is concerned with the problems of thinking, wants to realize what went on psychologically.

If we were to describe the process in the way of traditional logic, we would state numerous operations, like making abstractions, stating syllogisms, formulating axioms and general formulas, stating contradictions, deriving consequences by combining axioms, confronting facts with these consequences, and so forth.

Such a procedure is certainly good if one wishes to test each of the steps with regard to its logical correctness. Einstein himself is passionately interested in logical correctness, logical validity.

But what do we get if we follow such a procedure? We get an aggregate, a concatenation of a large number of operations, syllogisms, etc. Is this aggregate an adequate picture of what has happened? What many logicians do, the way they think, is somehow like this: a man facing a work of architecture, a fine building, focuses, in order to understand it, on the single bricks and also on the way in which they are cemented by the mortar. What he has at the end is not the building at all but a survey of the bricks and of their connections.⁵

In order to get at the real picture, we have to ask: How did the operations arise, how did they enter into the situation, what was their function within the actual process? Did they just drop in? Was the process a chain of happy accidents? Was the solution a consequence of trial and error, of mathematical guesswork? Why just these operations? No doubt there were other possibilities at some points. Why was Einstein moving in just this direction? How did it come about that after he made one step, he followed with just that other step?

I shall mention one specific point: How did the new axioms arise? Did Einstein just try any axioms of which certain ones then actually

happened to work? Did he formulate some propositions, put them together, and observe what happened until eventually he was fortunate enough to find a proper set? Did such propositions leap into the picture accidentally, and did the changes in the role, place, and function of the items, did their new interrelatedness appear merely as derived consequences?

The technique of axioms is a very useful tool. It is one of the most efficient techniques so far invented in logic and mathematics; a few general propositions provide all that is needed in order to derive the details. One can deal with a gigantic sum of facts, with huge numbers of propositions, by substituting for them a few sentences which in a formal sense are equivalent to all that knowledge. Some great discoveries in modern mathematics became possible only because this extremely simplifying technique was at hand. Einstein, too, used this tool in the accounts which he gave of his theory of relativity.

But, to repeat, the question for the psychologist is: Were these axioms introduced before the structural requirements,⁶ the structural changes of the situation were envisaged? Was it not the other way around? Surely, Einstein's thought did not put ready-made axioms or mathematical formulas together. The axioms were not the beginning but the outcome of what was going on. Before they came into the picture as formulated propositions, the situation as to the velocity of light and related topics had for a long time been structurally questionable to him, had in certain respects become inadequate, was in a state of transition. The axioms were only a matter of later formulation—after the real thing, the main discovery, had happened.⁷

⁶ In our discussions Einstein focused on the material content of the steps. He did not use the terms of the preceding sentences of the text, terms which follow from the structural approach of this book.

⁷ In this respect I wish to report some characteristic remarks of Einstein himself. Before the discovery that the crucial point, the solution, lay in the concept of time, more particularly in that of simultaneity, axioms played no role in the thought process—of this Einstein is sure. (The very moment he saw the gap, and realized the relevance of simultaneity, he knew this to be the crucial point for the solution.) But even afterward, in the final five weeks, it was not the axioms that came first. "No really productive man thinks in such a paper fashion," said Einstein. "The way the two triple sets of axioms are contrasted in the Einstein-Infeld book is not at all the way things happened

⁵ "I am not sure," Einstein said once in this context, "whether there can be a way of really understanding the miracle of thinking. Certainly you are right in trying to get at a deeper understanding of what really goes on in a thinking process. . . ."

When we proceed with an analysis in the sense of traditional logic, we easily forget that actually all the operations were parts of a unitary and beautifully consistent picture, that they developed as parts within one line of thinking; that they arose, functioned, and had their meaning within the whole process as the situation, its structure, its needs and demands were faced. In trying to grasp the structure of this great line of thinking, the reader may find himself at a loss in view of the wealth of events, of the breadth of the situation. What, then, were the decisive steps?

Let us recapitulate briefly.

First there was what we may call the foreperiod. Einstein was puzzled by the question, first, of the velocity of light when the observer is in motion. He considered, secondly, the consequences as to the question of "absolute rest." Thirdly, he then tried to make one alternative workable (is the velocity of light in Maxwell's equations a variable?), and obtained a negative result. There was, fourth, the Michelson experiment which confirmed the other alternative-and, fifth, the Lorentz-Fitzgerald hypothesis, which did not seem to go to the root of the trouble.

So far everything, including the meaning and structural role of time, space, measurement, light, etc., was understood in terms of traditional physics—structure I.

In this troubled situation the question arose: Is this structure itself, in which the Michelson result seems contradictory, really clear to me? This was the revolutionary moment. Einstein felt that

in the process of actual thinking. This was merely a later formulation of the subject matter, just a question of how the thing could afterwards best be written. The axioms express essentials in a condensed form. Once one has found such things one enjoys formulating them in that way; but in this process they did not grow out of any manipulation of axioms."

He added, "These thoughts did not come in any verbal formulation. I very rarely think in words at all. A thought comes, and I may try to express it in words afterward." When I remarked that many report that their thinking is always in words, he only laughed. I once told Einstein of my impression that "direction" is an important factor in thought processes. To this he said, "Such things were very strongly present. During all those years there was a feeling of direction, of going straight toward something concrete. It is, of course, very hard to express that feeling in words; but it was decidedly the case, and clearly to be distinguished from later considerations about the rational form of the solution. Of course, behind such a direction there is always something logical; but I have it in a kind of survey, in a way visually."

the contradiction should be viewed without prejudice, that the timehonored structure should be requestioned. Was this structure I adequate? Was it clear just with regard to the critical point—the question of light in relation to the question of movement? Was it clear in the situation of the Michelson experiment? All these questions were asked in a passionate effort to understand. And then the procedure became more specific in one step after another.

How was the velocity of light to be measured in a moving system?

How was time to be measured under these circumstances?

What does simultaneity mean in such a system?

But, then, what does simultaneity mean if the term is referred to

different places?

The meaning of simultaneity was clear if two events occur in the same place. But Einstein was suddenly struck by the fact that it was not equally clear for events in distant places. Here was a gap in any real understanding. He saw: It is blind simply to apply the customary meaning of simultaneity to these other cases. If simultaneity is to have a real meaning, we must raise the question of its factual recognition so that in concrete cases we can tell whether or not the term applies. (Clearly, this was a fundamental logical problem.)

The meaning of simultaneity in general had to be based on the clear simultaneity in the case of spatial coincidence. But this required that in every case of different location of two events the relative movement be taken into account. Thus the meaning, the structural role of simultaneity in its relation to movement under-

went a radical change.

Immediately, corresponding requirements follow for the measurement of time in general, for the meaning, say, of a second, and for the measurement of space, since they must now depend upon relative movement. As a result, the concepts of time-flow, of space, and of the measurement both of time and space changed their meaning radically.

At this point the introduction of the observer and his system of co-ordinates seemed to introduce a fundamentally arbitrary or subjective factor. "But the reality," Einstein felt, "cannot be so arbitrary and subjective." In his desire to get rid of this arbitrary element

and, at the same time, to get a concrete transformation formula between various systems, he realized that a basic invariant was needed, some factor that remains unaffected by the transition from one system to another. Obviously, both demands went in the same direction.

This led to the decisive step—the introduction of the velocity of light as the invariant. How would physics look if recentered with this as a starting point? Bold consequences followed one after another, and a new structure of physics was the consequence.

When Einstein reached the concrete transformation formula on the basis of this invariant, the Lorentz transformation appeared as a derivation—but now it was understood in a deeper, entirely new way, as a necessary formulation within the new structure of physics. The Michelson result, too, was now seen in an entirely new light, as a necessary result when the interplay of all relative measurements within the moving system was taken into account. Not the result was troublesome—he had felt that from the very beginning—but the behavior of the various items in the situation before finding the solution. With the deeper understanding of these items the result was required.

The picture was now improved. Einstein could proceed to the question of experimental verification.

In the briefest formulation: In a passionate desire for clearness, Einstein squarely faced the relation between the velocity of light and the movement of a system; confronted the theoretical structure of classical physics and the Michelson result.

A part-region in this field became crucial and was subjected to a radical examination.

Under this scrutiny a great gap was discovered (in the classical treatment of time).

The necessary steps for dealing with this difficulty were realized. As a result, the meaning of all the items involved underwent a change.

When a last arbitrariness in the situation had been eliminated, a new structure of physics crystallized.

Plans were made to subject the new system to experimental test. Radical structural changes were involved in the process, changes with regard to separateness and inner relatedness, grouping, centering, etc.; thereby deepening, changing the meaning of the items involved, their structural role, place, and function in the transition from structure I to structure II. It may be advisable to explain once more in what sense Einstein's achievement meant a change of structure.

1) In the Michelson situation—as in classical physics generally—time had been regarded as an independent variable and, therefore, as an independent tool in the business of measurement, entirely separate from, in no way functionally interdependent with the movements that were involved in that observational situation. Accordingly, the nature of time had been of no interest with regard to the apparently paradoxical result.

In Einstein's thought there arose an intimate relationship between time-values and the physical events themselves. Thus the role of time within the structure of physics was fundamentally altered.

This radical change was first clearly envisaged in the consideration of simultaneity. In a way, simultaneity split in two: the clear simultaneity of events in a given place and, related to it, but related by means of specific physical events, the simultaneity of events in different places, particularly under conditions of movement of the system.

2) As a consequence, space-values also changed their meaning and their role in the structure of physics. In the traditional view they, too, had been entirely separated from, independent of time and of physical events. Now an intimate relation was established. Space was no longer an empty and wholly indifferent container of physical facts. Space geometry became integrated with the dimension of time in a four-dimensional system, which in turn formed a new unitary structure with actual physical occurrences.

3) The velocity of light had so far been one velocity among many. Although the highest velocity known to the physicist, it had played the same role as other velocities. It had been fundamentally unrelated to the way in which time and space are measured. Now it was considered as closely bound up with time- and space-values, and as a fundamental fact in physics as a whole. Its role changed from that of a particular fact among many to that of a central issue in the system.

Many more items could be mentioned which changed their

meaning in the process, such as mass and energy, which now proved to be closely related. But it will not be necessary to discuss further particulars.

In appraising these transformations we must not forget that they took place in view of a gigantic given system. Every step had to be taken against a very strong gestalt—the traditional structure of physics, which fitted an enormous number of facts, apparently so flawless, so clear that any local change was bound to meet with the resistance of the whole strong and well-articulated structure. This was probably the reason why it took so long a time—seven years—until the crucial advance was made.

One could imagine that some of the necessary changes occurred to Einstein by chance, in a procedure of trial and error.⁸ Scrutiny of Einstein's thought always showed that when a step was taken this happened because it was required. Quite generally, if one knows how Einstein thinks, one knows that any blind and fortuitous procedure is foreign to his mind.

The only point at which there could have been some doubt in this respect was the introduction of the constancy of light velocity in Einstein's general transformation formulas. In a thinker of lesser stature this could have happened through mere tentative generalization of the Lorentz formula. But actually the essential step was not reached in this fashion; there was no mathematical guesswork in it.

In late years Einstein often told me about the problems on which he was working at the time. There was never a blind step. When he dropped any direction, it was only because he realized that it would introduce ununderstandable, arbitrary factors. Sometimes it happened that Einstein was faced with the difficulty that the mathematical tools were not far enough developed to allow a real clarification; nonetheless he would not lose sight of his problem and would often succeed in finding a way eventually, in which the seemingly insuperable difficulties could be surmounted.

⁸ In Act III, when Einstein examined whether a particular alternative would work, he actually did try several procedures. But although these attempts did not lead to a solution, they were by no means blind. At that stage it was wholly reasonable to test such possibilities.